A Reformulation of Social Potential Fields for Navigation Algorithms of Intelligent Vehicles

Édgar Martínez García

Abstract

This manuscript details a mathematical framework for autonomous navigation in intelligent vehicles operating under an Intelligent Transportation System (ITS) scheme. The framework tackles 5 issues: 1) the reformulation of the Social Force Model (SFM) adapted to vehicles; 2) the integration of the SFM with a general motion model that represents motion in any inertial frame; and 3) the vehicle kinematic restrictions with fundamentals in the actuators' velocity control. Furthermore, the scheme involves 4) the sensory detection of either obstacles or local goals, and with such sensing features, adaptive numeric weights are formulated in order to affect the navigation function of the vehicle, while avoiding collision, but reaching its next goal destination. Finally, 5) the state position vectors are formulated in terms of the actuators (electric motors) velocity control model. Experimental outdoors and indoors results are presented to show the feasibility of the proposed framework.

Keywords: navigation; motion dynamics; kinematic constraints; social potential fields.

1 Introduction

The present approach presents a framework that includes the kinematics and motion dynamics model in continuous-time merged with a general model that reformulates the social potential fields to solve the motion-planning problem. However, the acceleration vector cannot mathematically be easily integrated because a descriptive motion model is usually unknown. Some works dealing with potential fields treat the motion as a particle making this approach an applicable model to most rolling-based locomotion systems, and do not include geometry of motion constrained by the vehicle kinematics. Other approaches assume point, point-like, circular or polygonal robot primitives [10]. The proposed framework only considers the vehicle initial pose, and its geometric model to be known; assuming also, we directly are able to control the wheels' rate speed. With the wheels' velocity and with the vehicle's contact points location
the proposed model provides the vehicle's angular rotational speed to estimate the vehicle-bearing rate. The SFM is reformulated as a general velocity-based motion framework that expresses internal and external causes and effects of motion. The speed control is based on a functional form of motors rotational speed rate, and the vehicle's physical size is the core to determine the vehicle yaw speed. With such basis the actual and posterior position vectors are formulated. The combined scheme allows any forward kinematics, since it depends on the locomotion vehicle's design, and a weighting factor yielded from multiple sensing features. Using the motion dynamic equations, motion is no longer purely geometric because directions are computed on acceleration components and unit vectors. A vehicle φ is represented in its local fixed inertial frame located at $x_\mu = (x, y)^T$, in actual time $t$, bearing to $\theta_t$ and rotating at angular velocity $\omega_t$. Usually, φ pursues attractive goal destinations, to be reached at future desired locations $x_{t+1} = (x, y)^T$. Local goals exert attractive accelerative forces $F_\gamma$. α defines any obstacle, which exerts repulsive accelerative forces $F_\alpha$. The Cartesian distance between two points is defined at instantaneous time $t$ by $\|\vec{v}_{\mu\alpha}\| = \|x_\mu - x_\alpha\|$. 

2 General navigation model

The reformulated social potential field, which is integrated in the proposed framework, uses constrained dynamics to solve the motion problem. The actual velocity vector $v_t$ in (1) is controlled with ideal magnitudes when the function term $h(\cdot) = 1$. The ideal velocity is denoted by $v^o = v^o(\cos(\theta_t) \sin(\theta_t))^T$. The real velocity vector $v_t$ at actual time $t$ is defined in (3), the real velocity model involves causes, effects, and random fluctuations.

$$v_t = (v_{t+1} - v^o_t)h \frac{v_{t, max}}{v_t} \quad (1)$$

The ideal velocity vector is non-stationary defined by $v^o_t = v^o u_t$, where $v^o$ is a constant reference linear speed, and $u_t$ is a direction vector leading towards a desired direction which is changing over time. The factor $\eta$ is the gain value that if adjusted, defines the control convergence of the speed. The model (1) controls the velocity peaks exceeding maximal allowable velocities. The control takes effect when the function value is in the range $0 < h(\cdot) < 1$, as modelled in equation (2). The general velocity model (1) recursively controls two aspects. Firstly, it is the real velocity fluctuating around the ideal velocity $v^o$ magnitude; and secondly, removing divergences of real velocity magnitudes overpassing a maximal allowable velocity value.

$$h \frac{v_{t, max}}{v_t} = \begin{cases} 
1 & \text{if } v_{t, max} \leq v_{t, max} \\
\frac{v_{t, max}}{v_{t, max}} & \text{otherwise} 
\end{cases} \quad (2)$$
Figure 1: Above: reference velocity control; Below: maximal allowable velocity control. With parameters: $v_x = 55 \text{ m/s}$ and $v_y = 65 \text{ m/s}$, orientation at $75^\circ$, $\eta = 0.125$, $v_0 = 35 \text{ m/s}$ and $v_{\text{max}} = 40 \text{ m/s}$.

The real acceleration $\frac{dv}{dt}$ is a function which will depend on two global acceleration components. One is the social potential vector $F_t = (f_x, f_y)$ expressing internal and external causes of motion. The another term is the general velocity model $a_t = (a_x, a_y)^T$ representing emerging accelerations effects in any inertial system (global $a^I$, or local $a^R$).

$$\frac{dv}{dt} = F_t \cdot a_t'$$

(3)

3 Social potential eld model

Social potentials eld functions (SPF) are analysed separately in this section, although part of the real motion model (3). The equation (4) models the SPF to
describe the acceleration vector \( \mathbf{F}_t \), compounded by internal \( \mathbf{F}_i \) and external \( \mathbf{F}_e \) causes of motion. Likewise \( \mathbf{F}_t \), the accelerative causes of the vehicle motion. The emergent motion behaviour is expressed in terms of an acceleration vector namely \( \mathbf{F}_t = (f_x, f_y)^T \), which is a result of the sum of all accelerations involved with different events.

\[
\mathbf{F}_t = \mathbf{F}_i + \mathbf{F}_e
\]  

(4)

The accelerative forces are weighted with adaptive numeric functions \( w(m_t, f_t) \) of the bearing locations within the sensors field of view as defined in equation (5).

\[
\mathbf{F}_t = \mathbf{F}_i + \sum_\alpha w(m_t, f_\alpha) f_\alpha + \sum_\gamma w(m_t, f_\gamma) f_\gamma
\]  

(5)

The repulsive and attractive effects affecting the vehicle’s behaviour accentuate the magnitudes of motion given in expressions (6) and (7), where \( x_t \) is a distance vector between a goal/obstacle and the actual vehicle \( \phi \). And \( m_t = (x_{t+1} - x_t) \) is a unit vector expressing the direction towards a next desired location \( x_{t+1} \). Thus, the repulsive acceleration effect according to \( w_t \),

\[
\mathbf{F}_t (m) = w(m_t, f_t) f_t
\]  

(6)

and the attractive acceleration effect,

\[
\mathbf{F}_t (m) = w(m_t, f_t) f_t
\]  

(7)

The influence of the weight \( w_t \) will depend on how parallel the sensing direction \( \psi_i \) of a goal/obstacle, and the actual acceleration \( f_t \) are. As it is described in expression (8). If the actual orientation of the vector acceleration \( f_t \) is approximately the same as the actual desired orientation \( m_t \), then no change of direction is required. It is expected that the orientation of a goal/obstacle sensed at \( \psi_i \) is approximately along the bearing of the next desired position. However, if \( m_t \) and \( \psi_i \) orientations are quite different, then the horizontal component \( f_t \cos \psi \) must be decreased by performing vehicle yaw changes.

\[
w_t = \begin{cases} 1 & m_t \parallel f_t \cos(\psi_i) \\ \gamma_t & \text{otherwise} \end{cases}
\]  

(8)

The influence of rotations that the vehicle must carry out is given by an influence term \( \gamma_t \) which is an average of the fusion of all multi-sensory observations. As it was established an instrumented vehicle with \( s_n \) different sensors \( i \), \( \gamma_t \) is valued within a range of values \( 0 \leq \gamma \leq 1 \), based on an effective angle of perception \( \psi_i \),
\[
\gamma_t = \sin \frac{1}{s_n} \sum_{i} \frac{\psi_i^t}{\omega} \tag{9}
\]

Where \( s_n \) is the total number of sensors involved in the perception of the objective (goal/obstacle), and \( (\psi_i^t)^n \) are the angles at which each sensor \( i \) detected the same objective. In fact, expression (9) gives a greater numeric weight to objectives located nearly along the longitudinal vehicle's axis (fixed-frame, at 90°), because such kinematic orientation is the preferred motion direction.

4 Inertial frames descriptive model of motion

Vehicle acceleration is described in different inertial frames, so that local/global navigation can be carried out. The acceleration vector is denoted by \( \mathbf{a}_t \), and in formulation (3) we described the real acceleration \( \frac{\mathbf{d}^2 \mathbf{v}}{\mathbf{t}} \). Let \( v_t^r \) and \( v_t^l \) be the right and left linear velocities respectively, so that the instantaneous linear velocity of the centroid is defined by stating the expression \( v_t = (v_t^r + v_t^l) / 2 \). By defining the velocity vector \( \mathbf{v}_t^R = (v_x, v_y)^T \) in the vehicle frame, the components \( XY \) represent the 2D plane of motion and is given by the expression (10),

\[
\mathbf{v}_t^R = v_t \begin{pmatrix} \cos(\theta_t) \\ \sin(\theta_t) \end{pmatrix} \tag{10}
\]

Where \( \theta_t \) is the cg angle of motion of actual orientation relative to the initial vehicle position. In (11) the velocity vector is transformed into a global inertial frame by a rotation using an Euler matrix \( \mathbf{R} \) with angle of rotation \( \psi_t \) as the angle between inertial frames,

\[
\mathbf{v}_t^I = \mathbf{R}_{Z}(\psi_t)\mathbf{v}_t^R \tag{11}
\]

Substituting variables and developing the matrix multiplication, an equivalent expression is given by (12),

\[
\mathbf{v}_t^I = \begin{pmatrix} \cos(\theta_t) & \sin(\theta_t) \\ -\sin(\theta_t) & \cos(\theta_t) \end{pmatrix} \begin{pmatrix} v_t \cos(\psi_t) \\ v_t \sin(\psi_t) \end{pmatrix} \tag{12}
\]

Now, (12) is developed and the new expression into the global inertial frame becomes as expressed by (13),

\[
\mathbf{v}_t^I = \begin{pmatrix} v_t \cos(\theta_t) \cos(\psi_t) \\ v_t \sin(\theta_t) \sin(\psi_t) \end{pmatrix} = \begin{pmatrix} v_t \cos(\theta_t) \cos(\psi_t) + v_t \sin(\theta_t) \sin(\psi_t) \end{pmatrix} \tag{13}
\]

Simplifying (13) by using trigonometric identities, the new equation (14) expresses general cg velocity behaviour without vehicles kinematic constraints,
\[ v'_t = v_t \frac{\cos(\theta + \psi)}{\sin(\theta + \psi)} \]  \hspace{1cm} (14)

Having previous expression in terms of acceleration, subsequently the system is derived. So that, the resulting equation in the global inertial frame becomes as in (15),

\[ a'_t = v_t(\cos(\theta + \psi) + v_t \frac{\cos(\theta + \psi)}{\sin(\theta + \psi)} \]  \hspace{1cm} (15)

We inversely transform the acceleration into the vehicle coordinate framework by,

\[ a''_t = R_z^{-1}(\psi)a'_t \]  \hspace{1cm} (16)

Without lost of generality, the resulting simplified mathematical expression is now written as in equation (17)

\[ a''_t = v_t(\cos(\theta + \psi) + v_t \frac{\cos(\theta + \psi)}{\sin(\theta + \psi)} \]  \hspace{1cm} (17)

Equation (15) is about the same as equation (17). In the former, rotation frames angle \( \psi \) does matter, while in the latter, we have in fact the same expression with no rotation frames. Therefore, in such case \( \psi = 0 \). Hereafter, only (15) may be referenced for any Cartesian rigid roto-translation just by defining \( \psi = 0 \). Results on using (15) the vehicle cg moves while decreasing its angle at constant rate initially from 90° up to 50°, drawing a curved trajectory. Blue circle curves depict local frame data, and red crosses plot the cg motion in a global frame for \( \psi = \pi/2 \).

5 Dynamics model derivation

According to (3) mentioned earlier, now we derive the dynamics of motion explicitly defined. The real acceleration \( \frac{dv}{dt} = F_t \) \( a''_t = 0 \) is integrated and presented in terms of velocities. Let us assume that,

\[ a''_t = F_t \]  \hspace{1cm} (18)

By substituting (15) and (4) in (3) with \( \theta = 0 \), we have the equivalent expressions,

\[ v_t(\cos(\theta + \psi) + v_t \frac{\cos(\theta + \psi)}{\sin(\theta + \psi)} = F'_t + F_t \]  \hspace{1cm} (19)
Subsequently, let us define the term $F_t$ in (20) as the vehicle internal motivation of motion, where $\delta = x_{t+1} - x_t$ is the distance between the actual vehicle location $x_t$ and the next desired position $x_{t+1}$. There is a relaxation time $\delta$ defining the time of acceleration changes taken between each increase/decrease of speed.

$$F_t = \frac{1}{\delta} \frac{v^o}{t} (x_{t+1} - x_t) \cdot v_t$$  \hspace{1cm} (20)

The ideal linear speed $v_o$ set a desired velocity in $xy$ components, and a vector of actual measured velocity $v_t$. Arranging algebraically, and dropping off the actual measured velocity $v_t$ of (19), we solve to have a model of motion that combines general mobility, and external dynamic constraints defined in (21). Furthermore, the velocity vectors involved exert motion toward goals $v_t$, and against obstacles $v_t$, integrated into the model as a real velocity vector.
\[ \dot{\nu}_t = -d_r(\dot{\theta} + \dot{\psi}) \begin{pmatrix} -\sin \theta_t \\ \cos \theta_t \end{pmatrix} - v_t \begin{pmatrix} \cos \theta_t \\ \sin \theta_t \end{pmatrix} + \sum_\alpha v_t^\alpha + \sum_\gamma v_t^\gamma + \frac{v_{t\alpha}}{\parallel \delta_t \parallel} (x_{t+1} - x_t) \]  

(21)

Where \( d_r = \tau v_t \) is defined as a short displacement during a relaxation time. Expression (20) involves the vehicle’s actual and posterior position vectors, and kinematics constraints, and will be solved in next sections.

6 Vehicle position vector model

Most commonly used techniques to compute the angular velocity are based on discrete odometry strategies. Requiring right and left wheels displacements to infer an averaged rotational speed, [1]. The vehicle position vector is expressed by \( x_t = (x, y)^T \) at time \( t_n \), the actual position vector is a summation of all estimated positions overtime with respect to a common inertial frame where its origin \(< 0, 0 >\) at time \( t_0 \) is defined at the vehicle initial position.

6.1 Vehicle kinematics

Next figure depicts \( \beta_1 \) as a stationary value in which front wheels are fixed along vehicle’s longitudinal axis. Solving for the vehicle’s angular velocity \( \omega_t \), involving geometric parameters of wheels contact points. The vehicle maximal swift ability will totally depend on the vehicle’s contact points separation, defined by its width \( W \) and length \( L \).

This scheme controls wheels velocity, instead of getting encoder displacements. Linear velocities are defined by \( v_{l,r} = \dot{\varphi}_l\varphi_{l,r}, \) where \( \dot{\varphi}_l \) is the wheel
instantaneous angular velocity with nominal radius $r$. According to (22), the difference of angular velocities $\dot{\theta}_t$ in both sides of the vehicle will directly impact the vehicle turns. The vehicle rotational velocity $\omega_t$ with respect to its geometric centre depends on its differential linear velocity $v_x = v_x^r - v_x^l$, and its distance $l$ to rotate around its geometric centre, as defined by,

$$ l = \frac{v_x}{\omega_t} $$

(22)

The distance $l$ is a constant value that geometrically is set by the vehicle’s size, as in expression (23),

$$ l = \frac{W^2 + L^2}{2} $$

(23)

On such basis we deduce,

$$ \cos(\ ) = \frac{W}{l} $$

(24)

As we are interested on calculus of the $X$-axis differential velocity $v_x$ to satisfy the equation (22), the following relation between the differential linear velocity, and its $x$-component is defined by

$$ \cos(\ ) = \frac{v_x}{v^r} = \frac{v_x}{v}; \quad v \cos(\ ) = v_x $$

(25)

The equations (24) and (25) are equivalent, thus we drop $v_x$ from next equation (26),

$$ \frac{v_x}{v} = \frac{W}{l} $$

(26)

Substituting $v_x$ into (22) and algebraically arranging we deduce the following expression,

$$ \omega_t = \frac{(v_x^r - v_x^l)W}{l(W^2 + L^2)} = \frac{r_t(l)}{l(W^2 + L^2)} = K(\ ) $$

(27)

For simplicity we may assume equal wheels nominal radius, thus let $K$ be a constant numeric value,

$$ K = \frac{rW}{l(W^2 + L^2)} $$

(28)

In (27), the vehicle’s angular velocity is directly controlled by the wheels rotational rate. Wheels linear speeds are defined by $v_t = r_t dt$, depending on actual rotational rates $\dot{\theta}_t$, assuming nominal radius $r_t = r_l$. 


\[
\mathbf{u}_t = \begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} \frac{r_r}{2} & \frac{r_l}{2} \\ \frac{r_l}{2} & \frac{r_r}{2} \end{pmatrix} \begin{pmatrix} r \\ l \end{pmatrix}
\]  \tag{29}

For simplicity, we will assume same wheels nominal radius \( r_r = r_l \), and the factor \( K \) as it was defined in (28). Wheels angular velocity is controlled by directly using commercially available speed drivers which use asymptotic functions as given by

\[
\dot{\phi}_t(\Omega) = \left( a + e^{-\Lambda \Omega} - \mu \right) - b
\]  \tag{30}

Where \( a \) and \( b \) are constants that adjust the non-linear angular velocity behaviour curve, \( \Lambda \) is the constant of fast asymptotic fall, \( \Omega \) is a digital control word which is associated with an angular speed given directly by a user program, and \( \phi \) is the central value of the velocity curve.

Solving for (30), we integrate the equation and obtain the next expression

\[
\int_{a}^{b} \dot{\phi}_t \, d\Omega = \phi_t = \left( a - b \right) \Omega + a \ln\left(1 + e^{\Lambda(\Omega)}\right)
\]  \tag{31}

We synthesize the vehicle angle of direction as we also deduce a formal positioning model equation in the vector form by (32),

\[
\mathbf{x}_{t_n} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \int_{t_0}^{t_n} \begin{pmatrix} v_0 + \int_{t_0}^{t} r_t \, dt \\ \int_{t_0}^{t} r_t \, dt \end{pmatrix} \cos\left( \theta_0 + K \int_{t_0}^{t} (\dot{r}_r - \dot{r}_l) \, dt \right) \, dt \\
\int_{t_0}^{t} r_t \, dt \sin\left( \theta_0 + K \int_{t_0}^{t} (\dot{r}_r - \dot{r}_l) \, dt \right) \, dt
\]  \tag{32}

Expression (32) uses the wheels angular velocity \( \dot{r}_r, \dot{r}_l \), and partially simplifying previous expression,

\[
\dot{t} = K \int_{t_0}^{t_n} (\dot{r}_r - \dot{r}_l) \, dt
\]  \tag{33}

Substituting (31) in (33) and algebraically solving

\[
\dot{t} = K \begin{pmatrix} a & b \end{pmatrix} r + \frac{a}{1 + e^{\Lambda(\Omega)}} \begin{pmatrix} a & b \end{pmatrix} \left( a - b \right) \ln\left(1 + e^{\Lambda(\Omega)}\right)
\]  \tag{34}

then,

\[
\dot{t} = K \begin{pmatrix} a & b \end{pmatrix} (r - l) + \frac{a}{1 + e^{\Lambda(\Omega)}} \frac{1 + e^{\Lambda(\Omega)}}{1 + e^{\Lambda(\Omega)}}
\]  \tag{35}

We assume similar approach as [9], in which a dynamic motion algorithm was derived from the robot’s general dynamics based on velocity. Therefore, we write the actual position vector as,
The final orientation \( \theta_t \) is obtained by an integration of vehicle’s bearing during the interval of vehicle displacement \([t_0, t_n]\), in which wheels rotation is controlled rather than collecting absolute odometry measurements. We reformulate the vehicles bearing by

\[
\theta_t = \theta_0 + K \int_{t_0}^{t_1} (\dot{\varphi}_r - \dot{\varphi}_l) dt (37)
\]

and solving for the instantaneous angle,

\[
\theta_t = \theta_0 + \Phi_t (38)
\]

Basically, all terms for estimating the vector \( x_t \) has been mathematically defined in terms of the vehicle’s size, and the functional form of motors, and such an approach can be implemented for practicality.

\[
x_{t+1} = x_t + \int_{t}^{t+1} \left( \frac{v_t}{m_t} \right) f_t(\delta \mu) dt (40)
\]

The present context proposes to alter the actual orientation \( \theta_t \) by deploying the weighted accelerative navigation function \( w_t(\cdot) f_t \) of equations (6) and (7). The fundamentals of the algorithm is focusing on the expression (8) describing \( m f_t \). Here, the dot product determines how perpendicular the velocity bearing with respect to a desired orientation \( m \) is. If \( m \) and \( f \) are approximately aligned, then it means that the velocity orientation is projected along the actual desired goal, so that altering direction is not required.

However, \( f_t \cos(\psi_t) \) is the acceleration magnitude along the horizontal axis (common frame) respect to objective angle \( \psi \). A very small value of \( f_t \cos(\psi) \),
signifies that practically no change in direction is required. If such magnitude is too large, an important correction in orientation must be established through \( \lambda \). \( \lambda \) is a weighting factor defining the bearing \( \phi \) towards the objective. Such objective (attractive or repulsive) is represented through sensor data features, the more sensors detect the same feature, the more certainty about the bearing objective will improve the weighting factor \( \lambda \). If \( \phi \) is very near, or along the vehicle heading axis (about 90\(^\circ\)), then \( \lambda = 1 \) (see equation (9)).

The actual accelerative force \( f_t \) is altered and defined as \( f'_t \), there is an objective angle correction \( (\phi - \theta_t) \), thus, the direction of \( \|f_t\| \) is rotated by (41)

\[
f'_t = R(\phi_t - \theta_t)f_t
\]  

where \( R(\phi - \theta_t) \) an Euler rotation matrix which corrects the acceleration bearing, and developing the expression, we now have,

\[
f'_t = \begin{pmatrix} f_x \cos(\phi_t - \theta_t) - f_y \sin(\phi_t - \theta_t) \\ f_x \sin(\phi_t - \theta_t) - f_y \cos(\phi_t - \theta_t) \end{pmatrix}
\]  

(42)

Simplifying in the vector form \( f'_t = (f_x, f_y)^T \), we define the new posterior position vector by

\[
x_{t+1} = \begin{pmatrix} x_t \\ y_t \end{pmatrix} + \int_t^t f'_t \, dt = \begin{pmatrix} x_t \\ y_t \end{pmatrix} + \int_t^t \begin{pmatrix} f_x \cos(\phi_t - \theta_t) - f_y \sin(\phi_t - \theta_t) \\ f_x \sin(\phi_t - \theta_t) - f_y \cos(\phi_t - \theta_t) \end{pmatrix} \, dt
\]

(43)
8 Reactive navigation

A potential function is a differentiable real-valued function. The value of a potential function can be viewed as energy and hence the gradient of the potential is force. The gradient is a vector which points in the direction that maximally increases the function (differentiable real-valued function). We use the gradient to define a vector field, which assigns a vector to each point in the manifold. A gradient vector field, as its name suggests, assigns the gradient of some function to each point. The potential function approach directs a vehicle as if it were moving in a gradient vector field. Gradients can be intuitively viewed as forces acting on a positively charged particle vehicle which is attracted to the negatively charged goal. Obstacles also have a positive charge which forms a repulsive force directing the robot away from obstacles. The combination of repulsive and attractive forces hopefully directs the robot from the start location to the goal location while avoiding obstacles. We mainly deal with first order systems, viewing the gradients as velocity vectors instead of force-vectors.

8.1 Repulsive potential function

Equation (44) is a general exponential-based potential field function. $f_t$ is a scalar that exhibits a general $1D$ repulsive potential behaviour. $u_o$ is a constant defining the acceleration amplitude. $R$ is a stationary value defining the territorial object region, or defined too as the asymptotic potential falling value,

$$ f_t = u_o^o R e^{x_o \cdot x_u} \frac{R}{x \cdot x} $$  \hspace{1cm} (44)

The denominator is determined by the factor $(R^{-1} \cdot x \cdot x)$ and defines the function to respond fast for situations where the actual vehicle has interaction with obstacles in short inter-spaces. Solving for its gradient operator with partial derivatives, $f_t = \frac{\partial f_t}{\partial x} - \frac{\partial f_t}{\partial y}$,

$$ \int f_t \cdot \sum_x f_t \cdot x = \frac{R u_o^o}{(x \cdot x)^{1/2}} e^{x_o \cdot x_u} R \left( \frac{1}{2R} \right)(x \cdot x)^{1/2} R \left( \frac{1}{2R} \right)(x \cdot x)^{1/2} e^{x_o \cdot x_u} \left( \frac{1}{2} \right)(x \cdot x)^{3/2} $$

Arranging and ordering the terms, the expression becomes,
Figure 5: Potential fields general model. Left: Attractive potential; Right: Repulsive potential.

\[
\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle = \begin{bmatrix}
    u_o(x) R(\rho(x) - \rho(\mu)) e^{\|\rho(x) - \rho(\mu)\|/R} (1 - R(\rho(x) - \rho(\mu))^3/2 - u_o (\rho(x) - \rho(\mu)) e^{\|\rho(x) - \rho(\mu)\|/R} (1 - R(\rho(x) - \rho(\mu))^3/2)
\end{bmatrix}
\]

Thus, algebraically arranging let us factorize the main factors,

\[
\int \frac{f}{x} \frac{f}{y} \sum = \frac{u_o R(x) e^{x_\alpha x_\mu} R}{u_o R(y) e^{x_\alpha x_\mu} R} \quad \frac{u_o (x) e^{x_\alpha x_\mu} R}{u_o (y) e^{x_\alpha x_\mu} R}
\]

\[
\int \frac{f}{x} \frac{f}{y} \sum = \frac{u_o (x) e^{x_\alpha x_\mu} R}{u_o (y) e^{x_\alpha x_\mu} R}
\]

then to facilitate let us use notation \( f_i \) instead, and let us define too

\[
\delta_{ij} = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j \end{cases}
\]
Finally, some terms of the derived equation may be substituted and simply expressed as,

$$f_t = u^o e^{\frac{x_\alpha - x_\mu}{R}} (\frac{1}{R} \frac{R}{x_y}) x \ x$$

(48)

Previous expression if is defined in terms of a velocity vector, then it is the term that partially satisfies the real velocity of equation (21), it means that

$$v_t = \int{ f_t \ dt}.$$  

8.2 Attractive potential function

Similarly to previous section, now the equation that controls the vehicle course attracted to a local goal is defined by the general equation,

$$f_t = u^o e^{\frac{\|x_\gamma - x_\mu\|}{R}} R$$

(50)

Unlike the repulsive function, the attractive potential model does not include a denominator establishing strong attraction effects, but a smoothing effect. Thus, solving for its gradient operator,

$$\langle \frac{\partial f_t}{\partial x}, \frac{\partial f_t}{\partial y} \rangle = \begin{bmatrix}
-\frac{u^o e^{\frac{\|x_\gamma - x_\mu\|}{R}} R}{2} \frac{R}{\|x_\gamma - x_\mu\| R^2} (x_\mu - x_\gamma) \\
-\frac{u^o e^{\frac{\|x_\gamma - x_\mu\|}{R}} R}{2} \frac{R}{\|x_\gamma - x_\mu\| R^2} (y_\mu - y_\gamma)
\end{bmatrix}$$

(51)

then by simplify the expression, it becomes

$$f_t = u^o e^{\frac{\|x_\gamma - x_\mu\|}{R}} R$$

(52)

Thus, arranging algebraically the 2D potential function becomes

$$f_t = u^o e^{\frac{\|x_\gamma - x_\mu\|}{R}} R$$

(53)

In order to show an experimental situation representing both, attractive $x_t$, and repulsive features $x_t$, next figure depicts a sequence of outdoor experimental results using vision under natural light conditions. The white triangles located along the roadway-middle are features that represent local attractive goals ( ), positioned at next desired positions $x_{t+1}$. While blue regions over lane lines represent repulsive objectives ( ). Moreover, the yellow feature points are used for localization and place recognition.
Figure 6: Outdoor experiments using vision-based feature extraction, red regions are navigable ways; blue regions are repulsive areas; white triangles define attraction areas for the weighting function $w(u, f_i^γ)$ which impacts the potential field $f_i^γ(δ)$.

8.3 Combining different potential functions

The social potential fields are natural variations of potential fields according the environment geometric representation that characterize potential functions by inverse-force laws, rather than defined navigation functions. In next figure, the vehicle is displaced vertically from coordinates $x_0^γ(t) = (0, 0)^T$ towards $x_n^μ(t) = (0, 100)^T$, where the attractive goal $γ$ is located at (red triangle). There exist two obstacles namely $α$ and $β$ located at $x_α = (15, 50)^T$ and $x_β = (-2, 20)^T$ respectively. From this trajectory, the $xy$ velocity rates were yield, but vehicle’s trajectory was not controlled in order to depict how acceleration components behave when passing near the obstacles. The vehicle parameters for this experiment are $v^o = 0.5 \text{ m/s}$ from 0 m to 100 m along the vertical $y$-axis.

Previous figure at top-right depicts attractive accelerations. As the distance $\|γ - μ\|$ is gradually getting shorter, the rate of motion behaviour is decreasing until the vehicle reaches the $γ$. In figure 8 middle left-right, the acceleration components yielded by the obstacles are depicted versus the distance respect the actual vehicle positions. It is worth noting in figures 8 down left and right, how along the $y$-axis the acceleration components evolve (blue dots $x$-component, and red crosses $y$-component), in particular at $y$-coordinate 20 m and 50 m to naturally avoid obstacles $β$, and subsequently $α$, and then return to the desired trajectory.

In the last figure, real experimental data are depicted based in relation to the velocity model (1), the continuous red line is the vehicle path-track smoothly
Figure 7: Vehicle initial position at $\mathbf{x} = (0 \ 0)^T$, goal destination at $\mathbf{x} = (0 \ 100)^T$, an obstacle at $\mathbf{x} = (15 \ 50)^T$, another obstacle at $\mathbf{x} = (20 \ 20)^T$.

controlled with a set of local goals established a priori by the human user $(20 \ 15) \ (10 \ 20) \ (5 \ 15)$. Additionally, by using the sequence of actual and projected position vectors $(\mathbf{x})^i_{0}$ which were collected during the navigation task, the environment was mapped with LIDAR data. This environment model evidences how the velocity-based algorithm may control the vehicle's trajectory, although, the concerning of this manuscript is to accomplish safe navigation in urban roadways, rather than accurate vehicle localization, we only depict that the position vectors approximately match with the expected positions reached by controlling the vehicle's actuators speed. Further, the influence of the potential fields were adjusted by tuning the stationary parameters for the experiments.

In the figure (right), acceleration magnitude fields versus Cartesian plane depicts how the directional fields behaves in the presence of one goal and one obstacle exerting accelerative force $\mathbf{F} + \mathbf{F}$. The peak at $< -5 \ 0 >$ is the repulsive directional field with a standard deviation of falling value around 4 m. The attractive point is located at $< 0 \ 0 >$ with an acceleration value
Figure 8: Left: Real controlled navigation trajectory using estimated position vectors \((x_t)\) and \((x_{t+1})\) for mapping. The red continuous line is the vehicle trajectory; blue regions are LIDAR scans registered in common Cartesian framework. Right: 3D potential fields combining \(F_\gamma + F_\alpha\), at \(\langle -5, 0 \rangle\) \(\gamma\) is located, and at coordinates \(\langle 4, 5 \rangle\) an obstacle is located.

beneath the rest of the points, the nearest the goal is detected, the lowest the acceleration value is.

9 Conclusions

This work has presented a framework formulated for electric vehicles using a reactive navigation scheme for urban roadways for applications where autonomy is very critical. The proposed model provides practical solutions to a wide variety of dynamic scenarios and complex navigation problems, allowing easy implementation for real-time intelligent navigational tasks. The framework accepts as input the state vector of a robotic platform, and merges social potential fields, vehicle kinematics with explicit locomotion restrictions to find inverse kinematics, and a general motion dynamic model. The presented scheme fuses two main equations, one that models the effects of a generalized motion of a particle (vehicle’s cg) involving different inertial frames; and another equation that models causes of motion dynamics, including internal and external. The proposed approach includes an analysis of the vehicle’s geometric constraints in continuous-time kinematics to establish an equation that models the actuators velocity control, and the vehicles angular velocity in terms of its physical size, rather than using odometric information. The author considered two different potentials models used into an urban scenario with co-existence of static and dynamic obstacles. As the robot follows the gradient of the potential field, it moves toward the goal. Social potential fields alter the navigation function such that the resulting motion reflects free-collision navigation leading toward a goal destination. In addition, the potential forces that exert navigation control
over the vehicle are enhanced by using a weighting function that computes and adaptive numeric factor to weight the vector of acceleration forces, acting the vehicles direction and velocity. The proposed framework has been experimentally demonstrated by combining algorithms indoors and outdoors with real vehicles, and vehicle-like robotic platforms as well. The issues regarding potential fields were tested in vehicle-like robotic platforms, while vision-based, LIDAR sensing, and other sensor data were collected in the streets. This work presented simulation results in relation to the reactive navigation scheme, and experimental results about the sensory information combining visual perception of the roadway and limit lines; and the use of laser range finders to detect distances within a simulation program.

Bibliography


Edgar Martínez-García (edmartin@uacj.mx)
Laboratorio de Robótica, IIT,
Universidad Autónoma de Ciudad Juárez,
Av. Del Charro 450 Norte, Ciudad Juárez, Chih., México.
C.P. 32310, A.P. 1594-D.